WORKSHEET # IV

1. By using the definition of derivative, investigate whether the function $f(x) = |x-1|x^2 + \sin(x-1)$ is differentiable at $x = 1$ or not.

2. Using the definiton, calculate the derivatives of the following functions. Then evaluate the derivatives at the specified points.

   a) $f(x) = (x-1)^2 + 1 \; : \quad f'(-1)\,, f'(3)$

   b) $f(x) = \dfrac{1}{\sqrt{x}} \; : \quad f'(4)$

   c) $f(x) = \cos(x^2 - 1)$

3. Find the derivatives of the following functions.

   a) $y = \dfrac{x^3 + 7}{x}$

   b) $y = x^7 + \sqrt{7}x - \dfrac{1}{\pi + 1}$

   c) $y = (2x-5)(4-x)^{-1}$

   d) $y = \dfrac{(x^2+x)(x^2-x+1)}{x^4}$

   e) $y = (x^2+1)(x+5+\frac{1}{x})$

   f) $y = (\sec x + \tan x)(\sec x - \tan x)$

   g) $y = \tan(x + \cos x)$

   h) $y = \tan^2(\sin^3 x)$

   i) $y = \sec(\sqrt{x})\tan(\frac{1}{x})$

4. Find dy/dx for the following functions.

   a) $y = \cot\left(\dfrac{\sin x}{x}\right)$

   b) $y = \left(\dfrac{\sin x}{1+\cos x}\right)^2$

   c) $y = x^{-3}\sec^2(2x)$

   d) $y = \dfrac{\tan x}{1 + \tan x}$

   e) $y = \left(-1 - \dfrac{\csc\theta}{2} - \dfrac{\theta^2}{4}\right)^2$

   f) $y = (1-x)^4(1+\sin^2 x)^{-5}$

5. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent line is

   a) perpendicular to the line $y = 1 - \dfrac{x}{24}$,

   b) parallel to the line $y = \sqrt{2} - 12x$.

6. Find an equation of the normal line to the curve

$$x = 2\sec t \qquad y = \sqrt{3}\tan t \quad , \quad 0 < t < \frac{\pi}{4} \quad , \quad t = \frac{\pi}{6}$$

7. Find an equation for the tangent line to each of the following parametrized curves at the given value. Also, find the value of $\dfrac{d^2y}{dx^2}$ at the given point.

   a) $x = \sec^2 t - 1 \;,\; y = \tan t \;; \quad t = -\pi/4,$

   b) $x = -\sqrt{t+1} \;,\; y = \sqrt{3t} \;; \quad t = 3,$