Bifurcation Analysis With CROCKER Plots

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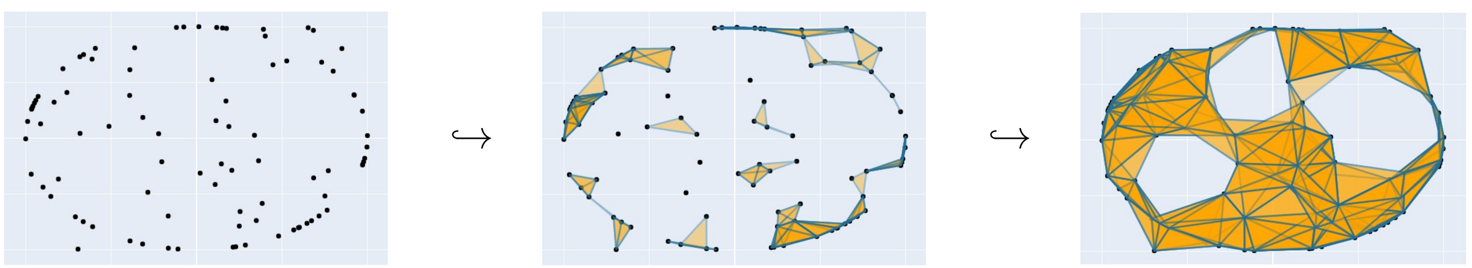
Introduction and Motivation

This study aims to address the question, *Can the CROCKER plots describe bifurcations in a dynamical system ?* Our proposed research has helped to answer this question by using zero and one dimensional Betti number. We show that the classical bifurcation method and the CROCKER plots do deliver the same topological information about a given dynamical system with interval parameters.

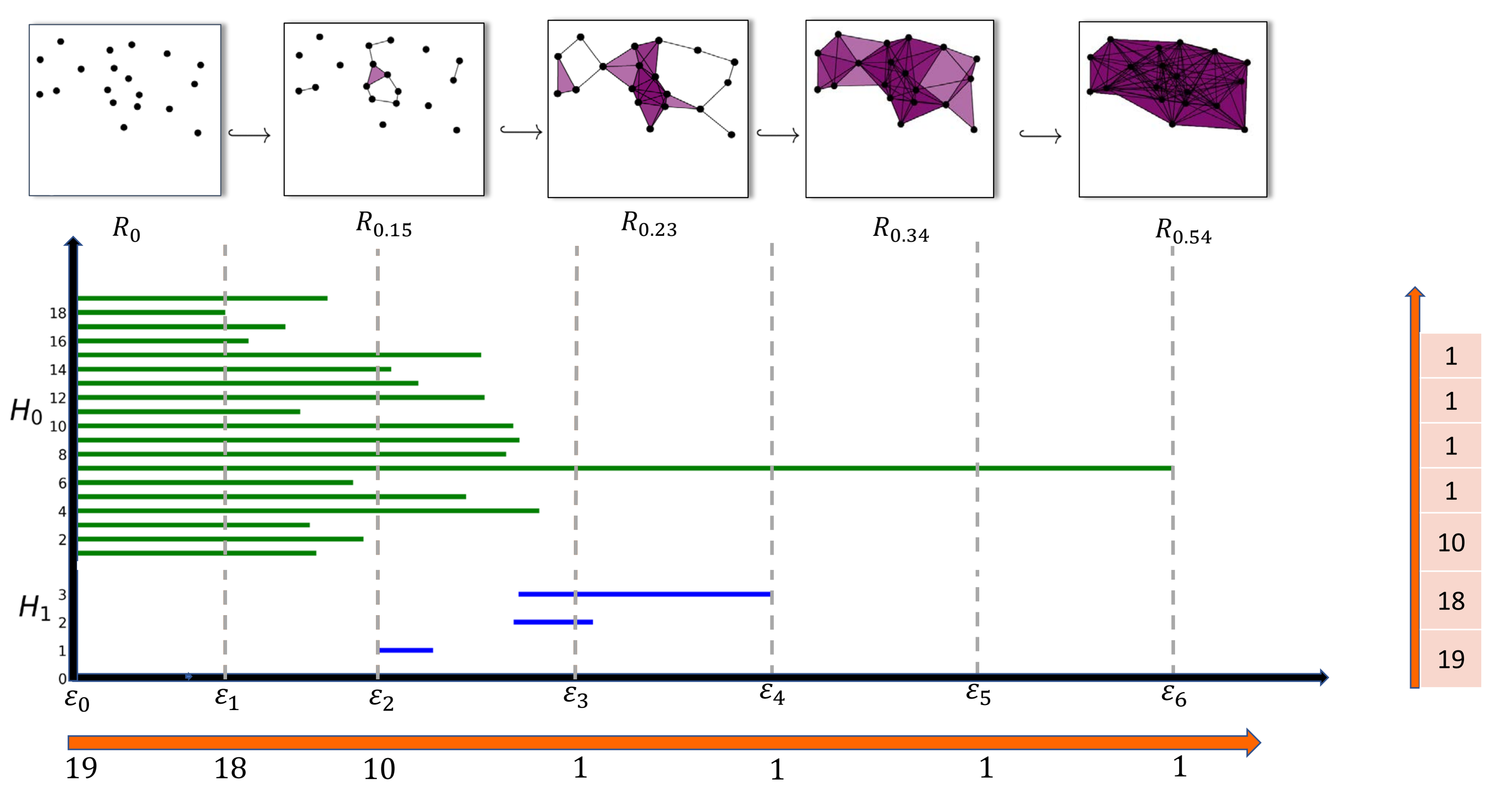
Vietoris-Rips Complexes and Filtrations

Given a point cloud D, the Vietoris-Rips is defined to be the simplicial complex whose simplices build on vertices that are at most ε apart.

$$R_\varepsilon(D) = \{\sigma \subset D \mid d(x, y) \leq \varepsilon, \text{ for all } x, y \in \sigma\}$$



Persistent Homology and Persistence Barcodes



CROCKER Plots

There is many vectorizations of persistence diagrams such as persistence landscapes, persistence images and persistent entropy, that make them amenable for machine learning tasks. In [3, 4], they introduce the Contour Realization Of computed k-dimensional hole Evolution in the Rips complex (CROCKER). This contour diagram consists of Betti numbers and varying time parameters.

Bifurcation Analysis

Bifurcations are qualitative alterations in the behavior of dynamical systems. As a result, detecting them is critical since they can indicate when a system is transitioning from normal operation to imminent breakdown.

Lorenz System

We consider a classical dynamical system called the Lorenz system [2] which is one of the earliest known examples of chaotic behavior. This system consists of three ordinary differential equations referred to as Lorenz equations:

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= x(\rho - z) - y, \\ \dot{z} &= xy - \beta z. \end{aligned}$$

where the constants σ, ρ, β are system parameters and x is proportional to the rate of convection, and y and z are the horizontal and vertical temperature variation, respectively.

Henon Map

In two dimensions, Henon [1] attempted to recreate the geometry of the Lorenz attractor. This requires the stretching and folding of space, which is accomplished with the discrete system that is now known as the Henon map:

$$\begin{aligned} x_{n+1} &= 1 - ax_n^2 + y_n \\ y_{n+1} &= bx_n \end{aligned}$$

where a and b are system parameters and the initial conditions are $x_0, y_0 = 0, 0$. If a $= 1.4$ and b$= 0.3$, The behavior of system which is called standard Henon map is chaotic.

References

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[3] Chad M. Topaz, Lori Ziegelmeier, and Tom Halverson.
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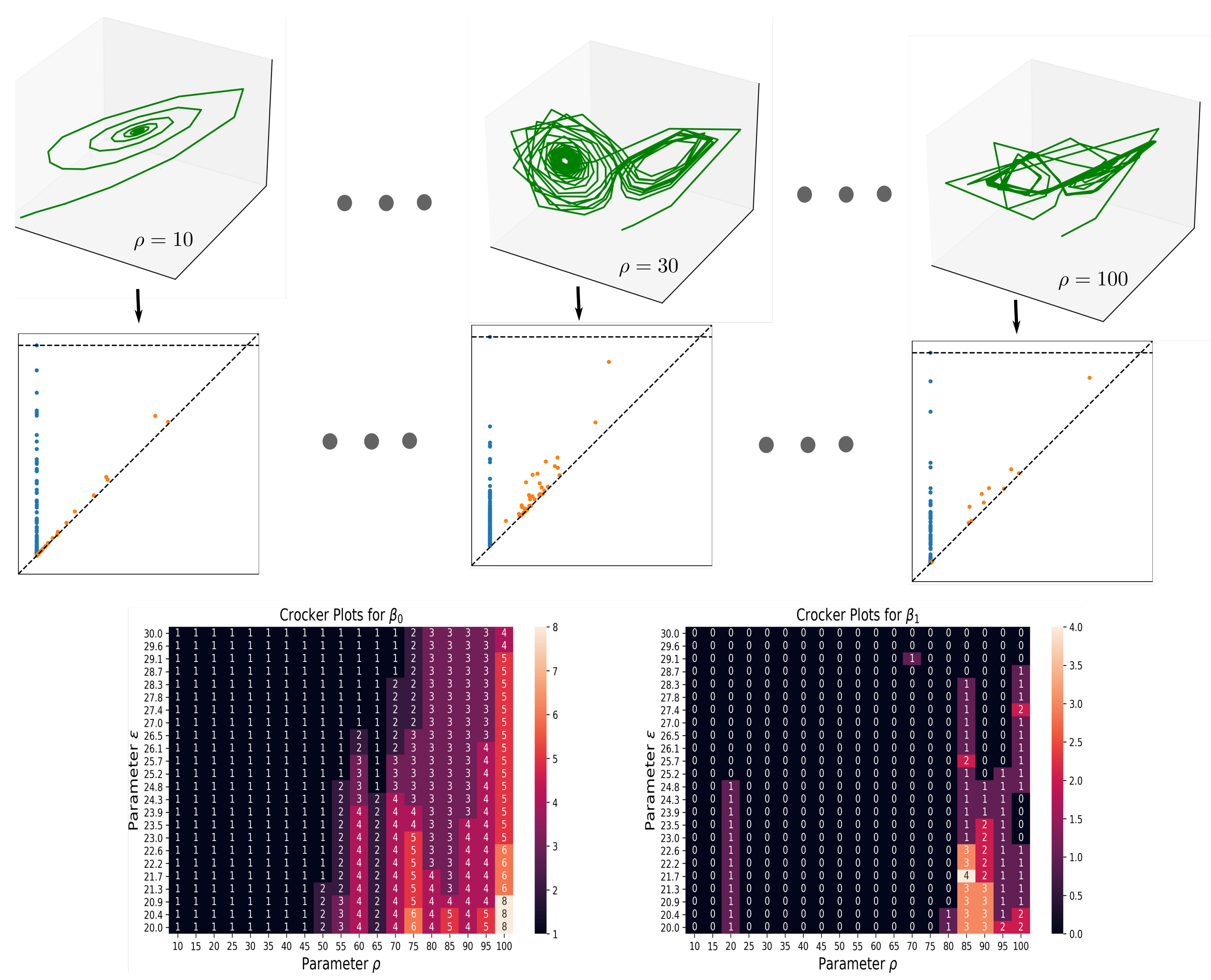
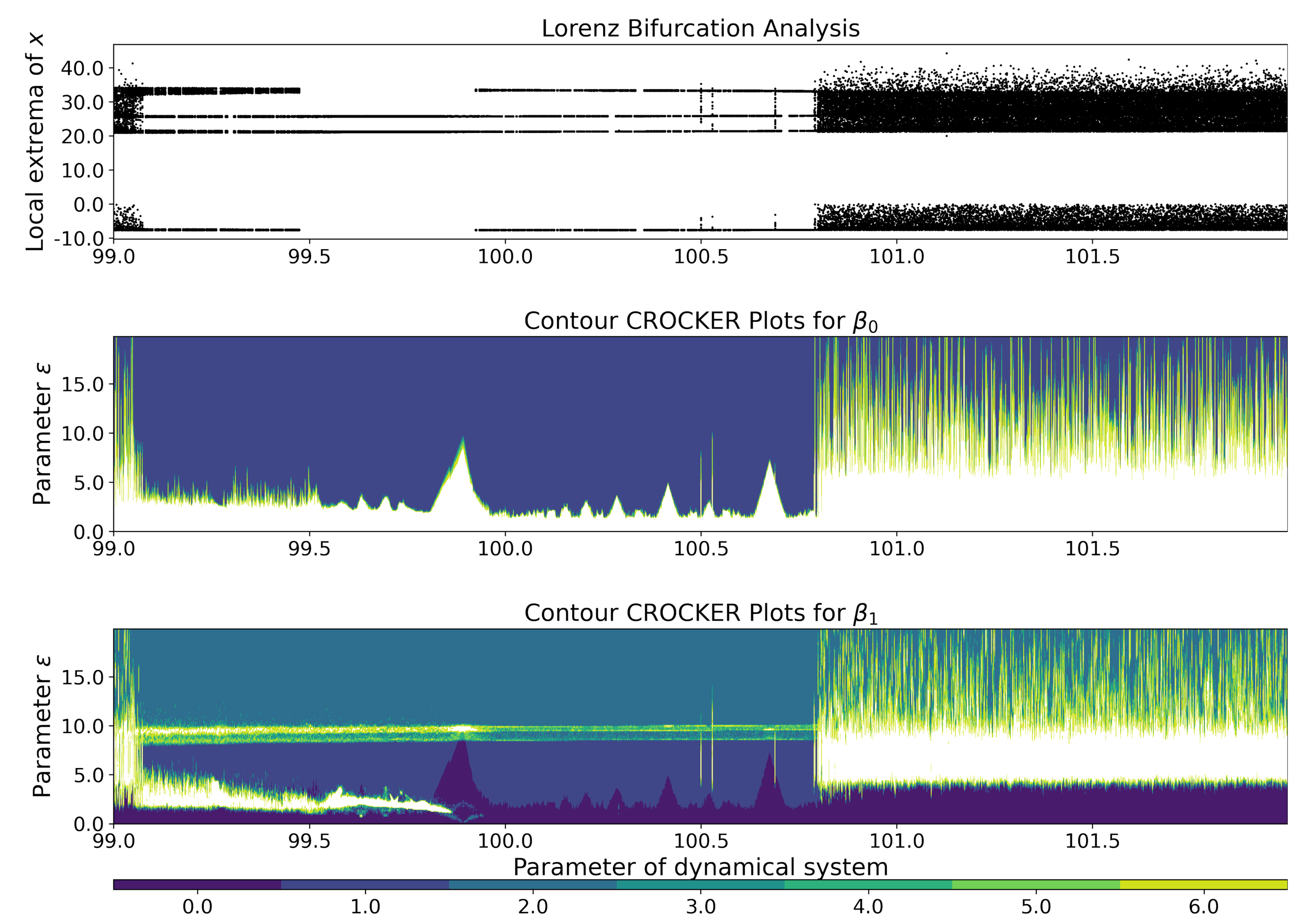
PLOS ONE, 14(3):1–18, 03 2019.

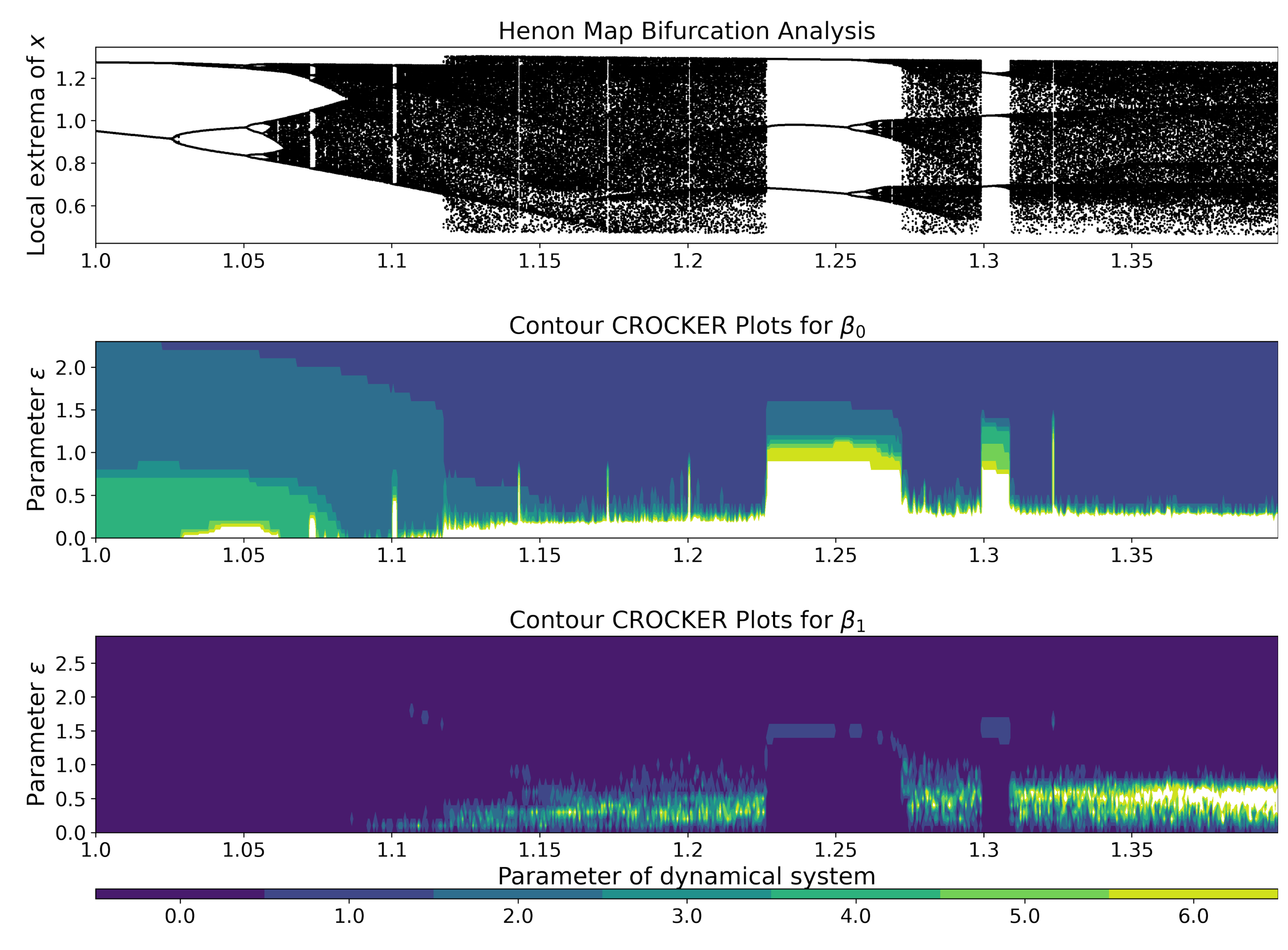
CROCKER of Lorenz System



Bifurcation of Lorenz System



Bifurcation of Henon Map



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